2. N. N. Pilyugin, S. L. Sukhodol'skii, and G. A. Tirskii, "Hypersonic radiating gas flow around a cone and wedge," in: Mathematical Modeling of Aerothermochemical Phenomena [in Russian], Vychisl. Tsentr Akad. Nauk SSSR, Moscow (1974).
3. G. G. Chernyi, Hypersonic Gas Flow [in Russian], Fizmatgiz, Moscow (1959).
4. A. F. Messiter, "Lift of slender delta wings according to Newtonian theory," AIAA J., 1, No. 4 (1963).
5. V. V. Sychev, "On hypersonic flow around slender bodies at high angles of attack," Dokl. Akad. Nauk SSSR, 131, No. 4 (1960).
6. A. V. Krasil'nikov, "On vibrations of slender bodies at high angles of attack in hypersonic flow," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 6 (1969).
7. A. I. Bolubinskii and V. N. Golubkin, "On the three-dimensional hypersonic gas flow around a slender wing," Dokl. Akad. Nauk SSSR, 234, No. 5 (1977).
8. A. I. Golubinskii and V. N. Golubkin, "On the theory of three-dimensional hypersonic gas flow around a body," Dokl. Akad. Nauk SSSR, 258, No. 1 (1981).
9. A. I. Golubinskii and V. N. Golubkin, "Three-dimensional hypersonic flow around a body of finite thickness," Uch. Zap. TsAGI, 13, No. 2 (1982).
10. V. I. Bogatko, A. A. Grib, and G. A. Kolton, "Hypersonic gas flow around a slender wing of variable shape," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 4 (1979).
11. S. I. Pai, Radiating Gasdynamics [Russian translation], Mir, Moscow (1968).

EQUATIONS OF MECHANICS OF A GAS-PARTICLE MIXTURE

A. P. Ershov

UDC 532.529

We consider the non-steady, one-dimensional motion of a gas containing suspended particles. For subsonic relative velocities of the gas and particles, the equations of the system have two complex characteristics [1] corresponding to instability of the solution to the Cauchy problem. The physical cause of the instability [2, 3] is a rise in the filtration velocity of the gas and a corresponding drop in pressure in regions where there is an increase in particle concentration. The pressure gradient encourages particle coagulation and perturbations grow exponentially. The rate of growth is inversely proportional to the wavelength of the perturbation.

It is important to be able to separate real physical flow instabilities from formal instability arising because of approximations in describing the mixture. An example of the latter is the rapid growth of short wavelength perturbations. In [3] an essential difference was pointed out between problems admitting steady motion of the phases (suspension of layer, precipitation of a suspension) from those of the non-steady type (passing of a shock wave through a suspension in gas). In the latter case, the velocity of relative motion of the phases goes to zero with time, and if the nonphysical fluctuations are removed, the Cauchy conditions can be correct. In the numerical solution of such problems, this is always understood.

In [3, 4] the random motion of the particles was considered as a stabilizing effect. In the present paper, we consider the non-steady-state problem at small particle concentrations, where the random motion of particles is not important. The equations obtained here include explicitly the interphase forces and the relative volume of the dispersed phase averaged over the volume of the particle. Thus the growth of short wavelength perturbations is suppressed.

1. Statement of the Problem and Preliminary Estimates. Following the treatment in [1, 3], we ignore the internal properties of the subsystem in the equations of mass and momentum and limit the discussion (as in [3]) to the case of a barotropic gas. Thus we do not have to deal with the energy equation. The system of equations has the form [3, 5]:

Siberian Branch, Academy of Sciences of the USSR, Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 79-87, November-December, 1983. Original article submitted August 8, 1982.

$$\partial\rho\varphi/\partial t + \partial\rho\varphi u/\partial x = 0,\ \rho_s(\partial\alpha/\partial t + \partial\alpha v/\partial x) = 0,$$
$$\rho\varphi(\partial u/\partial t + u\partial u/\partial x) + \varphi\partial p/\partial x = -nf_D,$$
$$\rho_s\alpha(\partial v/\partial t + v\partial v/\partial x) + \alpha\partial p/\partial x = nf_D,$$
$$\varphi + \alpha = 1,\ \alpha = nV,\ p = p(\rho), \tag{1.1}$$

where φ and α are the volume fractions of gas and particles, ρ is the gas density, ρ_s is the (constant) density of particles, u and v are the velocities of gas and particles, p is the gas pressure, n is the number concentration of particles, V is the particle volume, f_D is the interphase dissipative force, which includes viscous dissipation, ground resistance, and the associated mass force. Besides f_D, we consider the buoyancy force ($-V\partial p/\partial x$ for a single particle). We assume that inertial forces capable of causing prolonged steady motion are zero.

We will assume that f_D is a function of the flow variables. The associated mass force (involving derivatives) is ignored; this will be reasonable when $\rho_s \gg \rho$. The associated mass force does not qualitatively change the situation, as shown in [3]. The system (1.1) always has two real characteristics; for $\alpha \to 0$ they correspond to sound waves in the gas. If the relative velocity of the phases $w = u - v$ is subsonic, the other two characteristics are complex. For $\alpha \ll 1$ they can be written explicitly [1].

$$\lambda = dx/dt = v \pm i\beta w/\sqrt{1 - w^2/c^2}, \tag{1.2}$$

where c is the sound velocity in the gas, and $\sqrt{\alpha\rho/\varphi\rho_s} \ll 1$.

In the solution of the Cauchy problem, complex characteristics lead to exponential growth of perturbations. The characteristic growth time of a perturbation with wavevector k is

$$t_p \sim \sqrt{\frac{\varphi\rho_s}{\alpha\rho}}\frac{1}{kw}. \tag{1.3}$$

We estimate now the characteristic time for the equalization of the velocities of the two phases. We consider the simplest case, where all parameters are independent of x. Then from (1.1)

$$\partial w/\partial t = -nf_D(1/\rho\varphi + 1/\rho_s\alpha).$$

And from this we obtain an estimate for the equalization time

$$t_e \sim (R/C_D w)(\rho/\rho_s + \alpha/\varphi)^{-1}. \tag{1.4}$$

where R is the particle radius, and C_D is the resistance coefficient. If the streamlining of particles is not too great, $C_D \gtrsim 1$.

An initial perturbation grows markedly for $t_p \lesssim t_e$. Comparing (1.3) and (1.4), we find that the condition $kR \gg 1$ must be satisfied. Perturbations with wavelengths of order R or less are amplified. The rate of growth for short wavelength perturbations can be as large as desired, and the Cauchy conditions for (1.1) are incorrect.

In the numerical solution, an instability is observed if the step size of the computational net is of order R or less. Usually the opposite case occurs. For small step sizes, the instability is suppressed by introducing an artificial viscosity since (1.1) is not applicable for rapidly varying processes. It is also possible to smooth out fluctuations in the solution. According to [6], the perturbation growth of the smoothed (i.e., averaged over a region small compared with that characterizing the flow) α is bounded, and the Cauchy conditions for (1.1) are correct.

Another way of regularizing the solution within the framework of (1.1) is to refine the physical model used as the starting point. The random motion of particles leads to a pressure in the particle "gas" and to diffusion effects [3, 4]. Unlike the case of dense mixtures with prolonged steady motion considered in [4], in a dilute mixture a local equilibrium between the random motion of particles and the gas flow cannot be established.

The particles are streamlined by the gas and thus the particles experience hydrodynamical interaction forces which are transmitted through the surrounding gas. For a random particle distribution, the force reduces to the interaction of neighboring particles. According to [7], the interaction force between two spheres of radius R, separated by distance $l \gg R$ in an ideal fluid is given by $f_i \sim 2\pi\rho w^2 R^6/l^4$. We use this expression as a qualitative estimate of the actual force. Since f_i acts during the equalization time t_e of velocities of the phases, we estimate the resulting random velocity

$$v_c \sim w(R/l)^4(1 + \alpha\rho_s/\varphi\rho)^{-1} \leqslant w\alpha^{4/3}.$$

The pressure of the particle "gas" is given by $p_2 \sim \rho_s v_c^2 \sim \rho_s w^2 \alpha^{8/3}$. According to [3], taking into account p_2 as $\alpha \to 0$ does not insure stability because the exponent of α is greater than two.

The random displacement of a particle after time t_e is

$$l_c \sim v_c t_e \sim \frac{R\alpha^{4/3}\rho}{\rho_s}(\rho/\rho_s + \alpha/\varphi)^{-2} < R\alpha^{1/3} \ll R.$$

This gives an estimate of the wavelength over which diffusion effects are felt. For small enough α there exist waves that grow a finite, but large number of times after time t_e.

It is shown below that in the revised statement of the problem, a limit to growth appears even for long wavelengths of order R. Therefore, diffusion effects are not crucial in a dilute mixture. Below we will ignore the random motion of the particles.

2. Averaged Equations and Interphase Relations. In order to study the stability at short wavelengths, we need an equation applicable for rapidly varying flows. Normally, slowly varying motion of the fluid is considered, where the characteristic distance L is large compared to the separation between particles l. We can then choose a microscopic volume of linear dimension $\Delta x \gg l$ and average the equations of motion over this volume [8]. The validity of using averaged equations in describing rapidly varying motion, however, is not so obvious.

The required system of equations can be obtained by using a less well-known method of averaging. This is an average with respect to area, rather than volume. It is known that in a smooth, continuous flow, the average over a microscopic volume leads to the same results as an average over a microscopic area [8]. Indeed, the flow through the boundary of a reference volume is a surface-averaged quantity, and the equations of motion are closed by assuming the equivalence of volume and surface averages. A representative area with a linear dimension much larger than l will intersect the trajectories of a large number of particles, and a statistically meaningful average can be performed with respect to such an area. The form of the general conservation equations is not changed under surface-averaging [9].

However, the range of applicability of the surface-averaged quantities is wider. In the most instructive case of one dimension, averaging over an area normal to the direction of motion allows the study of sharp gradient flows because the rapid variation of the quantities perpendicular to the area does not affect the averaging procedure inside the area.

Thus in deriving the averaged equations, we can use a thin reference volume (Fig. 1) with a thickness Δx much less than l and possibly less than R. Those particles entirely within the reference volume can intersect its boundaries twice. The derivation is done in the usual way; below we present the equations with some comments.

For the solid phase, we ignore the random motion of particles, and it is convenient to consider the centers of mass of the particles as mass points. For a concentration n and a velocity v we have

$$\partial n/\partial t + \partial nv/\partial x = 0, \ \rho_s V(\partial nv/\partial t + \partial nv^2/\partial x) = nf, \tag{2.1}$$

where V is the particle volume, f is the force on a particle at position x and time t from the gas.

Quantities pertaining to the gas are averaged over a cross section of the gas phase, and the velocity u and u^2 are averaged with weight ρ. We let α and φ be the area fractions of the two-phase medium at the cross-section plane x = const. From Fig. 2, we can see that a particle whose center lies at a distance ξ from the reference plane will overlap by an area $A(\xi)$ dependent on the particle shape. Then α can be written as an integral over the particle cross section:

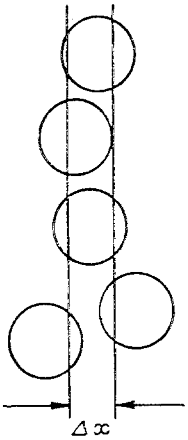

Fig. 1

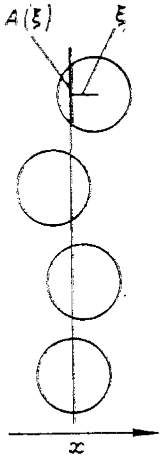

Fig. 2

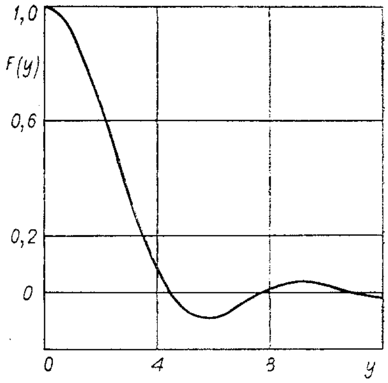

Fig. 3

$$\alpha(x, t) = \int_{-R}^{R} A(\xi)\, n(x + \xi, t)\, d\xi \qquad (2.2)$$

or more compactly as an integral over the particle volume V

$$\alpha = \int_V n dV, \quad \varphi = 1 - \alpha. \qquad (2.3)$$

If n is slowly varying, (2.2) and (2.3) reduce to $\alpha = nV$.

The equation of mass for the gas has the same form as in (1.1). We will write the equation of momentum for the medium as a whole, thus avoiding interphase forces inside the reference volume:

$$\frac{\partial \rho\varphi u}{\partial t} + \frac{\partial \rho\varphi \langle u^2 \rangle}{\partial x} + \frac{\partial}{\partial x}(p\varphi - \tau\alpha) + G = 0.$$

We let τ be the stress averaged over the particle cross section (for solid particles, a microscopic stress cannot be introduced, but the average over the cross section of each particle is completely determined by the action of the gas). Finally G is the Langrangian rate of change of momentum of the solid phase

$$G = \rho_s \left(\frac{\partial}{\partial t} \int_V nv dV + \frac{\partial}{\partial x} \int_V nv^2 dV \right).$$

This expression can be obtained in the same way as (2.2) and (2.3). Carrying the differentiation under the integral sign and using (2.1), we have

$$G = \frac{1}{V} \int_V nf dV. \qquad (2.4)$$

We introduce the gas velocity fluctuation $\delta u^2 = \langle u^2 \rangle - u^2$ and separate out the pressure (averaged with respect to the gas) from the stress in the solid phase: $\tau = -p - \tau'$. The sign of the correction τ' is chosen for convenience. After the usual transformations, we obtain the equations

$$\begin{gathered} \partial n/\partial t + \partial nv/\partial x = 0, \ \partial v/\partial t + v\partial v/\partial x = f/\rho_s V, \\ \partial \rho\varphi/\partial t + \partial \rho\varphi u/\partial x = 0, \\ \rho\varphi\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right) + \frac{\partial p}{\partial x} + \frac{\partial}{\partial x}(\rho\varphi\delta u^2 + \tau'\alpha) = -G. \end{gathered} \qquad (2.5)$$

With the help of (2.4), the momentum equation of the mixture takes on a form like the momentum equation for the gas, in which G appears as a force.

The system (2.5) can be used for rapidly varying motion if f, δu^2, and τ' are known. The usual relations for these quantities are applicable, with certain restrictions, only for slowly varying flow. Nevertheless, we can do some preliminary computations.

We write the microscopic stress tensor of the gas in the form

$$\sigma_{ik} = -p\delta_{ik} + \sigma'_{ik},$$

where σ' is near zero except near the particles. Then force f naturally separates into two parts

$$f = f_A + f_D = -\int_S p dS_1 + \int_S \sigma'_{1k} dS_k$$

where S is the particle surface. The average pressure p is formally defined in all space such that the buoyancy force f_A can be transformed using Gauss' theorem into an integral over the particle volume.

$$f_A = -\int_V \frac{\partial p}{\partial x} dV. \tag{2.6}$$

The dissipative force f_D arises from deviations of the pressure from the average value near the particles and from viscous stresses in the gas. For slowly varying flow, in which we neglect the associated mass force component of f_D, it is customary to consider these forces as functions of the average quantities.

However, there is no justification for assuming the same is true for rapidly-varying flow. Therefore (2.5) with the conventional expression for f_D is valid only for motion with a characteristic distance scale $L \gg R$ (but possibly $L < l$). It would be extremely difficult to find f_D for the general case.

Below we treat the simpler problem of stability of slowly-varying flow. If we apply a small short wavelength perturbation to the solution, it is reasonable to assume that f_D is basically determined by the slowly varying flow, since perturbations of f_D will be smoothed out because it is an integral over the particle surface. Therefore in treating stability, we assume that f_D depends on the gas parameters averaged over the particle. More detailed restrictions on f_D are considered in the next section.

3. Stability Analysis. The instability of (1.1) follows from (2.5) with the following assumptions:

1) the flow is slowly varying so that $\alpha = nV$, $f_A = -V\delta p/\delta x$, $G = nf = -\alpha\delta p/\delta x + nf_D$;
2) the dissipative force f_D is a known function of the flow parameters;
3) fluctuations in the gas velocity and additional stresses in the particles can be ignored.

The first assumption is the most essential and leads to a purely differential system of equations. We first take into account a correction of the form $\rho\varphi\delta u^2 + \tau'\alpha$ in the momentum equation, keeping the first two assumptions. The fluctuation δu^2 arises from streamlining of particles and can be written in the following form [8], assuming a dilute mixture and velocities much less than the speed of sound:

$$\delta u^2 = \delta_1(\alpha) w^2, \ \delta_1 \sim \alpha.$$

For velocities which are not too small, the additional stress in the particles is given by $\tau' \sim \rho w^2$; this is of the form of a pressure drop in the gas around the particle. We thus have

$$\rho\varphi\delta u^2 + \tau'\alpha = \delta(\alpha)\rho(u-v)^2, \ \delta(\alpha) \sim \alpha. \tag{3.1}$$

It is essential that $\delta(\alpha) > 0$ because on average $\tau' > 0$ (the particle is "squeezed" by the gas flow).

Analysis of the characteristics of system (1.1) with the modified momentum equation of the gas

$$\rho\varphi\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right) + \varphi\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\delta\rho(u-v)^2 = -nf_D$$

leads to the equation

$$[(u-\lambda)(u-\lambda+2\delta w/\varphi) - (c^2 + \delta w^2/\varphi)](v-\lambda)^2$$
$$= 2c^2\beta^2 w(v-\lambda)\delta/\alpha + c^2\beta^2[(u-\lambda)(u-\lambda+2\delta w/\varphi) + w^2 d\delta/d\alpha].$$

Taking into account that $\delta \sim \alpha$ and $d\delta/d\alpha \sim 1$ and positive, as before we have two complex characteristics. To first order in α they are given by

$$\lambda = v \pm i\beta w \sqrt{1 + d\delta/d\alpha} / \sqrt{1 - w^2/c^2}.$$

The imaginary part of λ, determining the growth constant of the instability, is of the same order as in (1.2). Thus the correction (3.1) does not change the situation; the small factor δ makes it unimportant in the balance of forces of the gas. Below, these effects will be ignored; they can become important only for $\alpha \sim \varphi$ or close to a surface of discontinuity where $\tau'\alpha$ and δu^2 change over a distance of order R and (3.1) becomes of order G on the right hand side of the momentum equation.

We show now that the basic difficulty is the assumption of slowly varying flow. System (1.1) is not valid for short wavelengths, and therefore any estimate of the perturbation growth factor based on these equations is incorrect. For example, the natural expression for the buoyancy force (2.6) is an integral over the particle volume of the pressure gradient; in (1.1) this force is $-V\partial p/\partial x$, the first term in a Taylor series. Obviously, (2.6) will smooth out short wavelength fluctuations. The same is true of the integral expressions for α and G.

We consider the stability of (2.3) through (2.6) in the general integrodifferential form. The unperturbed solution will be taken as slowly varying with a characteristic distance scale $L >> R$. The fluctuations δu^2 and τ' are ignored. The perturbation is denoted by a prime and is proportional to the factor $\exp(i(kx - wt))$. We consider short wavelengths such that $kL >> 1$ (kR can be of order unity). In the linear approximation, we obtain from (2.5)

$$i\omega n' = ik(n'v + v'n), \quad -(i\omega - ikv)v' = f'/\rho_s V,$$
$$-\rho\varphi(i\omega - iku)u' + ikp' = -G',$$
$$-\varphi(i\omega - iku)p' + ik\rho\varphi u' + (i\omega - iku)\rho\alpha' = 0. \qquad (3.2)$$

Because $kL >> 1$, terms involving derivatives of the unperturbed solution are omitted. We take the simplest case, $p' = c^2\rho'$. According to (2.2), the perturbation

$$\alpha' = n' \int_{-R}^{R} A(\xi)\, e^{ik\xi} d\xi.$$

For a spherical particle $A(\xi) = \pi(R^2 - \xi^2)$, and elementary integration leads to the expression

$$\alpha' = (4\pi R^3/3)n'F(kR) = Vn'F(kR), \qquad (3.3)$$

where F is the particle form factor

$$F(y) = 3(\sin y/y^3 - \cos y/y^2). \qquad (3.4)$$

F is shown in Fig. 3. For $y << 1$ we have $F \approx 1 - y^2/10$ and $F = O(y^{-2})$ for $y >> 1$. Thus, α is insensitive to long wavelength perturbations. The other integral quantities behave in a similar way.

We have from (2.4)

$$G' = \frac{n'}{V} \int_{-R}^{R} A(\xi) f(x + \xi)\, e^{ik\xi} d\xi + \frac{f'}{V} \int_{-R}^{R} A(\xi)\, n(x + \xi)\, e^{ik\xi} d\xi.$$

In the above integrals, we can evaluate the slowly verying functions f and n at the point x. Then

$$G' = (fn' + nf')F(kR). \qquad (3.5)$$

Finally, $f' = f_A' + f_D'$. The first term from (2.6) is elementary:

$$f_A' = -ikp'VF(kR). \qquad (3.6)$$

Unfortunately, the dissipative force cannot be expressed in a form applicable to short wavelength flows. Therefore, some arbitrariness in determining f_D' is unavoidable. If $f_D(\rho, w)$ is known for slowly varying flow, we can use the approximation

$$f_D' = f_\rho \rho' H(kR) + f_w (u' H(kR) - v'), \quad f_\rho = \partial f_D / \partial \rho, \quad f_w = \partial f_D / \partial w. \tag{3.7}$$

The formfactor $H(kR)$, whose exact form is not as yet determined, can suppress the dissipative force perturbation at short wavelengths; the effect is due to the surface-integral nature of f_D. We have $H(0) = 1$, and H possibly depends on the Reynolds and Mach numbers of the streamline flow. The velocity v' is constant, and its contribution is not suppressed at any value of k.

We use the notation $\lambda = \omega/k$, $f^* = f/ik\rho V$, $f_w^* = f_w/ik\rho V$, $f_\rho^* = f_\rho/ikV$ and introduce the small parameters $r = \rho/\rho_s$, $\gamma = nV/\varphi$. The system (3.2) and relations (3.3) to (3.7) lead to the dispersion equation

$$(\lambda - v)(\lambda - v - rf_w^*)\Big[(\lambda - u)(\lambda - u - \gamma HFf_w^*) - \Big(\frac{c^2}{\varphi} - \gamma F(c^2F - Hf_\rho^*)\Big)\Big] = r\gamma F\Big[(c^2F - Hf_\rho^*)((\lambda - u)^2 + f^* - f_w^*(\lambda - v)) - Hf_w^*(\lambda - u)\Big(\frac{c^2}{\varphi} + f^* - f_w^*(\lambda - v)\Big)\Big]. \tag{3.8}$$

If we take $F = 1$ and drop f^*, f_w^*, f_ρ^*, then (3.8) reduces to the characteristic equation for (1.1).

The right-hand side of (3.8) contains the small factor $r\gamma$, and for $kR \gg 1$, this goes to zero at least as fast as $(kR)^{-3}$ [for $H = O(1)$]. Therefore the roots of (3.8) will be close to those of the left-hand side, denoted by Λ_i. We have

$$\Lambda_1 = v, \quad \Lambda_2 = v + rf_w^* = v + f_w/ik\rho_s V.$$

The propagation velocity of these waves is close to the particle velocity. The imaginary part of Λ_2 corresponds to damping of the perturbation (for $f_w > 0$). This inequality can be violated only in a narrow critical region. The growth factor of the perturbation $-rf_w/\rho V$ is of the smallness of r and is independent of k. After the particle is in the critical region a finite time, the growth of the perturbation will be bounded, and it begins to damp out with decreasing w. This is a physical instability; fast-moving particles in the critical region are decelerated more weakly than slow-moving ones, and an initial particle velocity fluctuation grows.

The roots $\Lambda_{3,4}$ of the square brackets on the left hand side of (3.8) correspond approximately to sound waves in the gas; $\mathrm{Re}\Lambda_{3,4} \approx u \pm c$. The imaginary parts, to first order in γ, are

$$\mathrm{Im}\,\Lambda_{3,4} = \gamma FH(f_w^* \pm f_\rho^*/c)/2.$$

With decreasing w (practically even for $w \sim c$) the first term in the parentheses dominates. Therefore the growth factor will be of order $-\gamma FHC_D w/R$. It is seen from (1.4) that after a time t_e, the growth factor becomes of order unity.

When $w = 0$, f_w is always positive. From the physically reasonable condition that the state of rest of both phases be stable for all k, we obtain restrictions on the formfactor H. For $w \to 0$, $K(kR)F(kR) \geq 0$. This is always satisfied if $H = F$; then one must average the gas density and velocity in the expression for f_D over the particle volume in a similar fashion to the other quantities discussed above. This choice is made mostly in the interests of simplicity.

A more accurate calculation of the roots (denoted by λ_i) is done by evaluating the right-hand side of (3.8) (denoted by D below) at Λ_i. The corrections to $\Lambda_{3,4}$ are proportional to γr and are therefore not significant. Because of the closeness of $\Lambda_{1,2}$ to each other, $\lambda_{1,2}$ are more complicated:

$$\lambda_{1,2} = v + rf_w^*/2 \pm \sqrt{(rf_w^*/2)^2 + D/(w^2 - c^2)}.$$

For large kR the first term under the square root sign dominates because then D is small; this case has already been discussed. When $kR \sim 1$ or $\gamma > r$, the second term can be ignored. Then $\mathrm{Im}\,\lambda_{1,2} \lessgtr w\sqrt{\gamma r}(kR)^{-3/2}$ and after the equalization time the growth of the perturbation will be finite, as before.

When $|w| \approx c$ the three roots of the left hand side of (3.8) are close in value and the above approximations cannot be used. The term involving D is more significant when the three roots are practically equal. Then $\lambda_{1,2,3} = \Lambda_{1,2,3} + (D/2c)^{1/3}$ and the imaginary part is bounded. Because D is small, the wave growth will also be bounded. In the strongly nonsteady case, $|w|$ decreases with time and $|w| \approx c$ is possible for each particle during a time small compared to t_e.

4. Discussion of the Results. In our model, a short wavelength perturbation can grow only by a finite extent. Therefore an initial slowly varying flow subjected to a small perturbation will change only slightly, and the Cauchy conditions will be correct.

Equations (2.3)-(2.6) can also be used in describing discontinuous flows. The stability of this system requires a separate analysis. However if the discontinuity is spread out over a distance of order R, it can be considered as continuous in our model; however, in this case the functions n and f cannot be taken out from under the integral sign in the definition of G'. This leads only to an insignificant change in the formfactor of (3.5) and all results concerning the short wavelength behavior remain in force. It should be pointed out that the physical relevance of the model over distances of order R will be determined by how realistic the force f_D is.

Instead of an integrodifferential system, at first glance it would appear to be simpler to use a differential approximation like (1.1). This can be done by expanding the slowly varying functions in the integral expressions (2.3), (2.4), (2.6) in Taylor series and keeping the first nonvanishing terms. This will lead to a purely differential system with derivatives with respect to the coordinates up to the third order. However, this system also leads to difficulties. In the stability analysis, this approximation is equivalent to replacing the formfactor F(kR) by its long wavelength asymptotic value $1 - (kR)^2/10$ and this leads to unbounded growth of perturbations for $kR \gg 1$. It is obvious that a differential approximation of any order will be unstable for large enough k and the corresponding Cauchy problem will be incorrect. This situation is different in principle from that in the gasdynamics of a single phase.

We briefly discuss the generalization of our method to slowly varying motion in three dimensions. The definitions of α, f_A, and G can easily be written in invariant form as integrals over the particle volume without singling out a particular direction in space. The flow variables can be taken as volume-averaged; they will then be averaged over a plane perpendicular to the propagation direction of the perturbation. One expects in the three-dimensional case that the smoothing out of short wavelength fluctuations insures the correctness of the Cauchy problem.

For very small particle concentrations, one can treat the motion of the gas independently and then compute the motion of the particles as acted upon by the gas. In our paper, we have considered the case where the volume content of particles α is small, but not neglgible. Our approach ignores the random motion of the particles and hence the natural microscopic parameter is the particle radius R. The random displacement of a particle l_c must be much smaller than R and thus the pressure $p_2 \ll p$, ρw^2. It follows from the discussion of Sec. 1 for the equalization time [see Eq. (1.4)] that the random motion can be ignored for $\alpha \lesssim 10^{-2}$ to 10^{-3}. In dense systems, or in cases where the time (1.4) is long, random motion of particles will be significant.

LITERATURE CITED

1. A. N. Kraiko and L. E. Sternin, "Theory of flow for a two-velocity continuous medium with solid or liquid particles," Prikl. Mekh., 29, No. 3 (1965).
2. S. V. Iordanskii and A. G. Kulikovskii, "On the motion of a fluid containing small particles," Izv. Akad. Nauk Mekh. Zhidk. Gaza, No. 4 (1977).
3. L. A. Klebanov, A. E. Kroshilin, et al., "On the stability of the Cauchy problem for the system of equations describing two-velocity flow in a two-phase medium," Prikl. Mekh., 46, No. 1 (1982).
4. O. V. Voinov and A. G. Petrov, "Stability of relative motions of phases in a two-phase medium," Zh. Prikl. Mekh. Tekh. Fiz., No. 1 (1982).
5. Kh. A. Pakhmatulin, "Basic gasdynamics of interpenetrating motion of continuous media," Prikl. Mekh., 20, No. 2 (1956).
6. A. N. Kraiko, "On the correctness of the Cauchy problem for a two-fluid model of a mixture of gas and particles," Prikl. Mekh., 46, No. 3 (1972).
7. O. V. Voinov, "On the force acting on a sphere in a nonuniform flow of an ideal incompressible fluid," Zh. Prikl. Mekh. Tekh. Fiz., No. 4 (1973).

8. R. I. Nigmatulin, Fundamentals of the Mechanics of a Heterogeneous Medium [in Russian], Nauka, Moscow (1978).
9. R. Panton, "Flow properties from the continuum viewpoint of a nonequilibrium gas—particle mixture," J. Fluid Mech., 31, Pt. 2 (1968).

MEASUREMENT OF CONDITIONALLY AVERAGED TURBULENCE CHARACTERISTICS IN THE PLANE WAKE BEHIND A CYLINDER

A. A. Praskovskii UDC 532.517.4;532.525.2

Methods describing turbulent flows using equations for probability density distribution (PDD) for velocity and concentration fluctuations are being actively developed [1, 2] in recent times. Such an approach to the study of turbulence is especially fruitful for the analysis of flows with chemical reactions. In formulating the closure of the equations for PDD, certain hypotheses based on the physical characteristics are used that require experimental verification. In particular, in obtaining the closure of equations for PDD of velocity fluctuations [1] on the basis of the Kolmogorov—Obukhov theory [3], it has been hypothesized that in a turbulent flow, turbulent energy dissipation measured at a constant value of velocity, does not depend on this value. Measurements of the dispersion of the streamwise velocity gradient in the plane of symmetry in the wake behind a circular cylinder where the flow is fully turbulent have demonstrated the correctness of this hypothesis [4]. The objective of the present paper is to verify the hypothesis given in [1] in those regions of turbulent flow where the intermittency coefficient is different from one, while the results of the measurements of the dispersion of the time derivative of streamwise velocity are used to estimate turbulent energy dissipation. During the measurements, a number of other conditionally averaged turbulent flow characteristics have been obtained which are of independent interest and some of them are also presented in this paper.

1. Measurements were made in the plane wake behind a circular cylinder of diameter d = 36 mm at a relative distance x/d = 38.6 behind the cylinder. The cylinder was mounted at a nozzle section of diameter 1200 mm in a wind-tunnel with open test-section, the free-stream turbulence in the absence of the cylinder was 0.4% at the nozzle section and 0.6% at the measuring section. Tests were conducted at a velocity $U_0 = 5.24$ m/sec, which corresponds to a Reynolds number $Re = U_0 d/\nu = 1.26 \cdot 10^4$, where ν is the kinematic coefficient of viscosity. Constant-temperature hot-wire anemometer DISA 55A01 with the transducer 55A22 using platinized tungsten wire, 5 μm in diameter and 1 mm long, was used to measure streamwise mean velocity component U and velocity fluctuations u(t), where t is the time. The output signal was recorded in the measuring ChM magnetometer "MR 800A Labcorder" in the frequency range 0-5 kHz, the recording time for each frame was 45 sec. The recorded realizations were passed through filters with a characteristic slope of 48 dB/octave and the lower and upper frequency bounds $f_l = 1$ Hz and $f_u = 800$ Hz, respectively, and then in frequency sampling of analog—digital converter $f_0 = 5$ kHz they were fed to a computer where their statistical characteristics were computed. The limitation of the frequency range of fluctuations in the high-frequency region made it possible to ensure a signal-to-noise ratio of 39-43 dB, but led to a reduction in the values of dispersion of the velocity gradient (quantitative estimates are given below) in the tests, and the energy spectrum of fluctuating velocities rapidly falls with an increase in frequency. Hence there is always a certain frequency, approximately equal to 2 kHz in the given experiment, at which the spectral density of the signal and noise are equalled and above which the noise exceeds the signal. At 800 Hz, the signal level was an order of magnitude higher than the noise and it determined the choice of the frequency limit for the filter. It is worth noting that in these experiments the basic source of noise was the magnetograph whose characteristic dynamic range at $f_u = 5$ kHz was approximately 37 dB.

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 87-94, November-December, 1983. Original article submitted September 30, 1982.

0021-8944/83/2406-0843$07.50